**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Sol: As the service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off the mean time increases by 10 min , so 45+10=55min

-µ)/(SD)

Given=60,µ=55,SD=8

z= 0.625

P(X<=60)

p=stats.norm.cdf(abs(0.625)) = 0.7340

the probability that the service manager cannot meet his commitment:

compute P(X>60)= 1- P(X<=60) =1-0.7340 = 0.26598

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Sol: find P(38<X<44)= P(X<44) – P(X>38)

-µ)/(SD)

Givenµ=38,SD=6,n=400

Z1= (44-38)/6= 1

Z2= 38-38/6 =0

P(X<44) =stats.norm.cdf(1) = 0.84

P(X>44)=1-0.84 = 0.16 =16%

P(X>38)= 1-stats.norm.cdf(0)= 0.5

P(38<X<44)= P(X<44) – P(X>38)=0.84-0.50=0.34= 34%

As 16% < 34%

Hence, FALSE

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Sol: P(X<30)

Z=(30-38)/6 = -1.33

stats.norm.cdf(Z)= 0.091 = 9%

Calculate 9% of total employees

400 \* (9/100) =36

Hence , TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Sol: From the properties of random variable:

The mean remains same for both 2 *X*1 and *X*1 + *X*2  i.e 2µ, while the variance for 2 *X*1 is 4σ^2 and for *X*1 + *X*2  is 2σ^2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Sol: From the above details,we have to excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Z value at 0.5th percentile is given as

                                         Z(0.005) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

                         Z(0.995) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

      a = -(20\*2.576) + 100= 48.5

      b = (20\*2.576)+100= 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Sol:

Given ,Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) follows(mean, sd^2)

Mean1(m1)=5, Mean2(m2)=7,

Standarddev1(SD1)=3, Standarddev2(SD2)=4

Total Mean =m1+m2= 5+7=12

Total SD =sqrt(SD1^2+SD2^2)=sqrt(9+16)=5

Total mean profit in rupees= 12\*45= Rs.540 billion

Total SD profit in rupees= 5\*45= Rs.225 billion

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Sol : stats.norm.interval(0.95,540,225)

Rupee range= [99.00810347848784, 980.9918965215122] in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company
2. Which of the two divisions has a larger probability of making a loss in a given year?

Sol:

Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3) = 0.0477

Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)=0.04005

The first **division** of **company**, thus have **larger probability**of making a loss in a given year.